Exercises

4.1 Exercise

Quad picture tree. QPT
a. What is a disadvantage of the QPT?
b. Give an equation for a measure of region uniformity. Explain why you think it is a good measure of uniformity. Explain how this measure would be used to determine if a region should be split in the QPT image splitting procedure.
c. Compute the uniformity measure for the upper left quad of the data. Show details of the calculation. Explain why the quad should or should not be split.
d. Compute the quad picture tree, QPT, corresponding to a reasonable segmentation of the given data. Use a region splitting method. Start with the entire image. Explain your splitting criterion. The uniformity calculations need not be shown in detail. Make your explanation clear.

3	2	2	1	1	1	1	1
5	3	2	2	1	1	1	1
7	6	3	3	2	1	1	1
7	7	5	4	3	1	1	1
8	7	5	4	3	1	1	1
8	8	5	3	2	1	1	1
8	8	6	4	2	1	2	1
8	8	8	5	1	1	1	1

Image Data

4.2 Exercise

Quad picture tree. QPT
a. 5 points. What is a disadvantage of the QPT?
b. 10 points. Give an equation for a measure of region uniformity. Explain why you think it is a good measure of uniformity. Explain how this measure would be used to determine if a region should be split in the QPT image splitting procedure.
c. 10 points. Compute the uniformity measure for the lower right quad of the data. Show details of the calculation. Explain why the quad should or should not be split.

0	0	7	8	7	7	7	7
0	1	7	7	7	7	7	7
0	0	7	7	7	7	7	7
1	0	5	8	7	7	7	7
1	8	8	1	7	7	7	7
8	8	8	1	7	7	7	7
8	8	8	8	8	7	8	8
8	8	8	8	8	8	7	8

Solution

0	0	7	8	7	7	7	7
0	1	7	7	7	7	7	7
0	0	7	7	7	7	7	7
1	0	5	8	7	7	7	7
1	8	8	1	7	7	7	7
8	8	8	1	7	7	7	7
8	8	8	8	8	7	8	8
8	8	8	8	8	8	7	8

R1 R2

7	7	7	7
7	7	7	7
8	7	8	8
8	8	7	8

R3 R4

$$\mu_1 = 7$$
$$\mu_2 = 7$$
$$\mu_{31} = 7.75$$
$$\mu_4 = 7.75$$
$$\sigma_1^2 = 0$$
$$\sigma_2^2 = 0$$
$$\sigma_3^2 = \frac{.25^2 + .75^2 + .25^2 + .25^2}{4} = \frac{.065 + .065 + .065 + .56}{4} = \frac{.755}{4} = .189$$
$$\sigma_4^2 = \frac{.25^2 + .75^2 + .25^2 + .25^2}{4} = \frac{.065 + .065 + .065 + .56}{4} = \frac{.755}{4} = .189$$

$$F = \frac{\sum_i \#(R_i)\sigma_i^2}{\#(R)} = \frac{4(0 + 0 + .189 + .189)}{16} = \frac{2 * .189}{4} = \frac{.189}{2} = .0945$$

4.3 Exercise

Compute the quad picture tree, qpt, corresponding to a reasonable segmentation of the given data. Use a region splitting method.
a. Give a criterion for when a region should be split based upon the variance of a region. Give the equation.
b. Compute the variance for the upper left quad of the data.
c. Compute the qpt for the segmentation of the data. start with the entire image. Explain your splitting criterion. The variance calculations need not be shown in detail. Make your explanation clear.

0	0	7	8	9	9	9	9
0	1	7	7	9	9	9	9
0	0	9	9	9	9	9	9
1	0	5	8	9	9	9	9
1	0	0	1	0	9	1	1
0	0	0	0	3	5	1	1
0	1	0	0	11	12	5	5
0	0	0	1	11	11	4	5

Image Data

4.4 Exercise

Quad picture tree. QPT
a. What is a disadvantage of the QPT?
b. Give an equation for a measure of region uniformity. Explain why you think it is a good measure of uniformity. Explain how this measure would be used to determine if a region should be split in the QPT image splitting procedure.
c. Compute the uniformity measure for the lower left quad of the data. Show details of the calculation. Explain why the quad should or should not be split.
d. Compute the quad picture tree, QPT, corresponding to a reasonable segmentation of the given data. Use a region splitting method. Start with the entire image. Explain your splitting criterion. The uniformity calculations need not be shown in detail. Make your explanation clear.

0	0	7	8	7	7	7	7
0	1	7	7	7	7	7	7
0	0	7	7	7	7	7	7
1	0	5	8	7	7	7	7
1	8	8	1	7	7	7	7
8	8	8	1	7	7	7	7
8	8	8	8	8	7	8	8
8	8	8	8	8	8	7	8

Solution

measure of uniformtiy

$$F = \frac{\sum_i \#(R_i)\sigma_i^2}{\#(R)}$$

0	0	7	8	7	7	7	7
0	1	7	7	7	7	7	7
0	0	7	7	7	7	7	7
1	0	5	8	7	7	7	7
1	8	8	1	7	7	7	7
8	8	8	1	7	7	7	7
8	8	8	8	8	7	8	8
8	8	8	8	8	8	7	8

1	8	8	1
8	8	8	1
8	8	8	8
8	8	8	8

$$\mu_1 = \frac{1+8+8+8}{4} = 6.25$$

$$\sigma_1^2 = \frac{(1-6.25)^2 + (8-6.25)^2 + (8-6.25)^2 + (8-6.25)^2}{4} = \frac{27.56+.3.01+.3.01+.3.01}{4} = \frac{36.59}{4} = 9.14$$

$$\mu_2 = \frac{1+8+1+8}{4} = 4.5$$

$$\sigma_2^2 = \frac{(1-4.5)^2 + (8-4.5)^2 + (1-4.5)^2 + (8-4.5)^2}{4} = \frac{12,25+.12.25+.12.25+12.25}{4} = \frac{49}{4} = 12.25$$

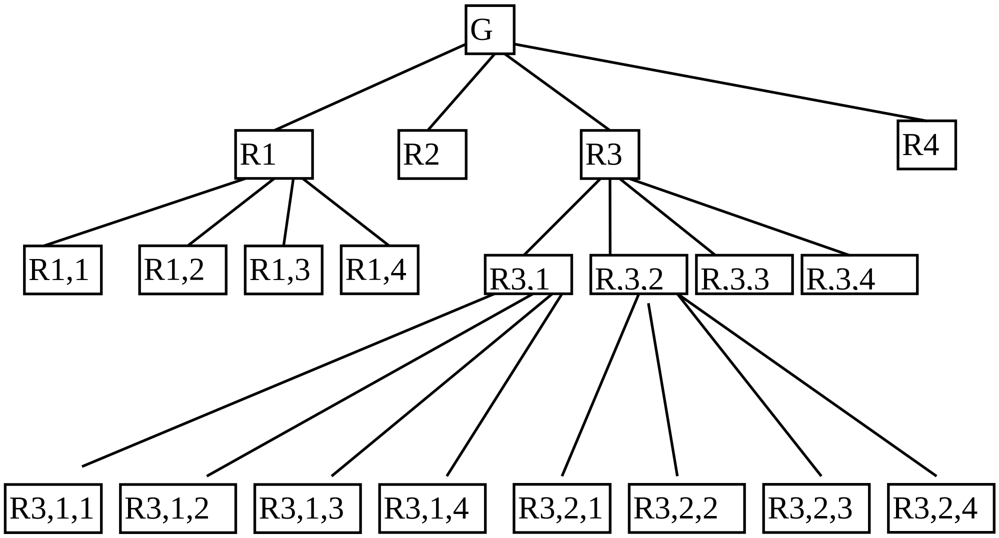
$$\mu_3 = \frac{8+8+8+8}{4} = 8$$

$$\sigma_3^2 = 0$$

$$\mu_4 = \frac{8+8+8+8}{4} = 8$$

$$\sigma_4^2 = 0$$

$$F = \frac{4*9.14 + 4*12.25 + 4*0 + 4*0}{16} = \frac{85.56}{16} = 5.35$$



4.5 Exercise

a. Give a measure of uniformity to determine when two regions should be split.
Explain why you think it is a good measure.

b. Apply your measure to the top four regions of the quad-tree decomposition of the following data. Explain whether these regions should remain separate or should they be merged.

Columns 1 through 12

126	126	124	123	124	127	124	125	127	130	135	138
127	125	125	126	127	126	124	125	129	133	139	144
125	125	125	126	125	125	127	129	133	137	141	144
123	122	124	126	127	125	129	132	135	139	142	144
122	122	124	127	127	127	129	131	135	139	142	142
123	123	125	127	128	130	129	132	135	140	141	141
123	123	125	127	128	129	130	131	134	135	137	137
123	122	124	125	127	127	128	128	127	126	125	123
122	122	121	124	125	124	125	123	120	116	113	111
114	114	118	121	122	121	123	118	111	105	102	100
115	114	116	118	119	121	109	108	105	100	90	77
114	114	114	114	114	114	110	97	87	82	78	76
111	112	113	113	107	102	101	87	72	72	82	91
106	109	113	112	100	88	73	80	80	78	79	77
105	108	112	107	91	78	70	85	88	68	43	29
111	111	107	97	84	73	76	69	57	38	9	2

Columns 13 through 16

141	143	148	152
148	151	148	147
147	148	151	150
145	145	149	148
142	142	144	143
141	141	140	139
134	135	133	131
122	123	119	116
109	109	105	103
93	83	83	87
75	80	85	94
84	99	105	106
92	88	83	79
57	33	22	24
21	16	23	17
21	52	71	42

Solution.

U_i = 4.65, 24.89, 49.43, 141,96

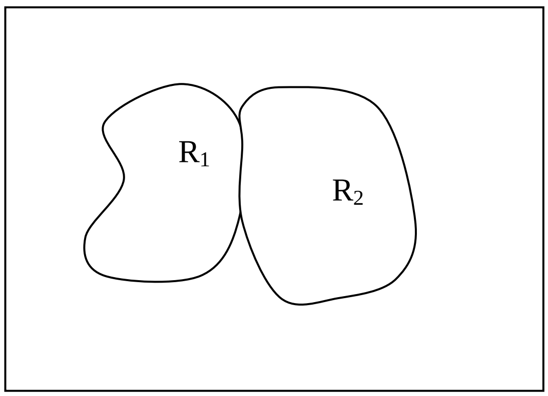
σ_i^2 = 6.17, 68, 227, 932

4.6 Exercise

Given a gray-scale image function g(x,y) and two adjacent regions R_1 and R_2.

a. Region merging based upon gray-level values g(x,y) will always give the wrong regions associated with real objects in certain situations. Give an example of one of these situations. What is the basic assumption about the objects for region merging based upon gray-level values?

b. Describe a specific method to determine if the two regions should be merged to form a larger region. Give equations, formulas, and all details in the calculations.



c. For the following data apply your method, from b, and determine if the regions should be merged.

The region R_1 consists of all the pixels above the thick black line.

The region R_2 consists of all the pixels below the thick black line.

R_1

135	140	141	141	141	141	140	139
134	135	137	137	134	135	133	131
127	126	125	123	122	123	119	116
120	116	113	111	109	109	105	103
111	105	102	100	93	83	83	87
105	100	90	77	75	80	85	94
87	82	78	76	84	99	105	106
72	72	82	91	92	88	83	79

R_2

Solution

One simple test could be to consider the region R_i uniform if $|\mu - \mu_i| < \varepsilon$. If all the regions were uniform then R would not be split into the subregions.

Another measure of uniformity is $U_1(R) = \frac{\sum_{i=1}^{4} \#(R_i)\sigma_i^2}{\#(R)}$ where #(R) is the number of pixels in region R. A smaller $U_1(R)$ corresponds to a more uniform region R. If the σ_i^2 are small, then this indicates that the split region will be more uniform. If the region is nonuniform, $U_1(R)$ is large, then split the region.

Another measure is $U_2(R) = \frac{\sum_{i=1}^{4}(\mu_i - \mu)}{\sum_{i=1}^{4}\sigma_i^2}$. If $U_2(R)$ is large then R is not uniform and should be split into the subregions.

An additional measure of uniformity for regions is $U_3 = 1 - \sum_{R_j} \frac{w_j \sigma_j^2}{\sigma_{\max}^2}$. Here $\sigma_{\max}^2 = \frac{(g\max - g\min)^2}{2}$ where gmax and gmin are the max and minimum gray-level values in the regions and w_j is a weight associated with region R_j. Two regions have good separation if the between class variance is large and the within class variances are large. This indicates that the regions should not be merged. Therefore, if U_3 is large the regions should not be merged and a small U_3 indicates the regions should be merged.

The Fisher distance function is another measure of region uniformity. The equation $U_4 = \frac{(n_1 + n_2)(\mu_1 - \mu_2)^2}{n_1\sigma_1^2 + n_2\sigma_2^2} = \frac{n\sigma^2}{n_1\sigma_1^2 + n_2\sigma_2^2} - 1$ is the Fisher distance. If the value is low, the regions should be merged. The term n is the number of points in both regions while σ^2 is the variance of the combined regions. If σ_i^2 =0 for every i. then this means the regions are uniform and should not be merged. If the numerator is also zero, one would consider merging.

R_1 is the top region.
R_2 is the bottom region.

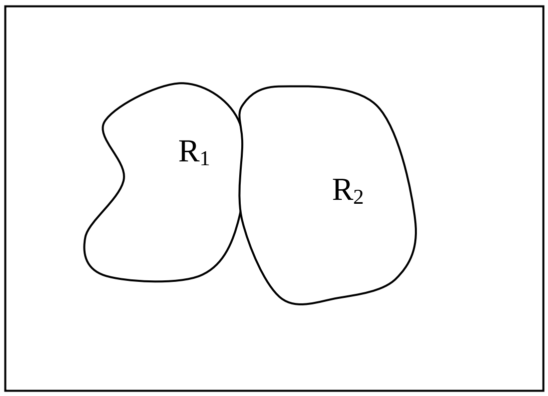
The mean of the combined regions is 107.8906. The standard deviation sigma is 22.0575.
The mean of region R_1 is 126.8438. The standard deviation of R_1 sigma is 11.7210.
The mean of region R_2 is 88.9375 . The standard deviation of R_2 sigma is 10.8280 .
The measure U_4 is 2.8221.
The regions should not be merged.

4.7 Exercise

Given a gray-scale image function g(x,y) and two adjacent regions R_1 and R_2.

a.

Describe in general the criterion one applies to determine if two regions should be merged.



b.

Explain why the region merging process is a low-level vision process. There is a basic assumption made that must be true for the region formation processes to give the correct regions. What is this assumption? Give an example where this assumption is violated.

c.

Compute the means and standard deviations for regions R1, R2, R4 in the supplied data.

d.

The Fisher distance function for two regions is $U_4 = \frac{(n_1 + n_2)(\mu_1 - \mu_2)^2}{n_1\sigma_1^2 + n_2\sigma_2^2}$.

Compute the Fisher distance for R2,R1 and R2,R4 . Interpret the results and draw any conclusions.

Indicate whether the regions should be merged.

e.

Compute the $Q_{ij} = \frac{(\mu_i - \mu_j)^2 n_i n_j (n_i + n_j - 2)}{(n_i\sigma_i^2 + n_j\sigma_j^2)(n_i + n_j)}$ distance for R2,R1 and R2,R4. Interpret the results and draw any conclusions. Indicate whether the regions should be merged.

R_1, R_2, R_3, R_4

122	122	121	124	125	124	125	123
114	114	118	121	122	121	123	118
115	114	116	118	119	121	109	108
114	114	114	114	114	114	110	97
111	112	113	113	107	102	101	87
106	109	113	112	100	88	73	80
105	108	112	107	91	78	70	85
111	111	107	97	84	73	76	69

Solution

a. One examines the two regions to determine if the combined regions have a uniformity better or equal to the uniformity of each separate region.

b. region formation is a low-level vision process because we are using only uniformity properties of the gray-levels to form the regions.

A basic assumption is that the gray-levels are uniform in the regions that correspond to objects. This assumption would be violated for an object with strong lighting variations across the object.

c.

$$\mu_1 = 117.2 \;\; \mu_2 = 117.1 \;\; \mu_3 = 109.2 \;\; \mu_4 = 85.3$$

$$\sigma_1 = 3.54, \sigma_1^2 = 12.5 \;\; \sigma_2 = 7.6, \sigma_2^2 = 57.7 \;\; \sigma_3 = 4.1, \sigma_3^2 = 16.5 \;\; \sigma_4 = 11.8, \sigma_4^2 = 139.2$$

d.

The Fisher distance is

$$U_4 = \frac{(\mu_i - \mu_j)^2 (n_i + n_j)}{(n_i \sigma_i^2 + n_j \sigma_j^2)}$$

for regions 2 and 1.

$$U_4 = \frac{(117.1 - 117.2)^2 (16 + 16)}{(16 * 57.7 + 16 * 12.5)} = \frac{3.2}{1123} = .0028$$

for regions 2 and 4

$$U_4 = \frac{(117.1 - 85.3)^2 (16 + 16)}{(16 * 57.7 + 16 * 139.2)} = \frac{1017.2}{3150.4} = .32$$

. This indicates that region 1 and 2 should be merged.

e. $$Q_{ij} = \frac{(\mu_i - \mu_j)^2 n_i n_j (n_i + n_j - 2)}{(n_i \sigma_i^2 + n_j \sigma_j^2)(n_i + n_j)}$$

$$Q_{21} = \frac{(117.1 - 117.2)^2 (16*16)(16+16-2)}{(16*57.7 + 16*12.5)(16+16)} = \frac{768}{35942} = .002$$

$$Q_{24} = \frac{(117.1 - 85.3)^2 (16*16)(16+16-2)}{(16*57.7 + 16*139.2)(16+16)} = \frac{7864320}{100813} = 78$$

Regions 1 and 2 should be merged