1. Exercises.

1.1 Exercise

For the given data

a. Consider the pixels as nodes on a graph. Compute the property difference in gray levels. Draw an arc between two nodes if the property is close. Draw the graph for the given data for the lines y=0,1,2.

b. Determine the different regions from the graph.

c. Explain the term region leakage and how it can occur.

3	2	2	1	1	1	1	1
5	3	2	2	1	1	1	1
7	6	3	3	2	1	1	1
7	7	5	4	3	1	1	1
8	7	5	4	3	1	1	1
8	8	5	3	2	1	1	1
8	8	6	4	2	1	2	1
8	8	8	5	1	1	1	1

1.2 Region growing considering the pixels as nodes on a graph

a. Give a criterion for measuring the closeness between two and connecting the nodes with an arc. Your methods should give you reasonable segmentation of the data .
b. Draw the graph for the given data based upon your criterion for connecting nodes with arcs.
c. Determine the different regions from the graph structure
d. Explain a common pitfall that can occur in region growing methods.

Image Data

y \ x	0	1	2	3	4	5	6	7
7	3	2	2	1	1	1	1	1
6	6	6	2	2	1	1	1	1
5	7	7	7	7	7	1	1	1
4	7	7	6	7	6	7	1	1
3	8	7	7	8	6	7	1	1
2	8	8	7	3	2	1	1	1
1	8	8	8	4	2	1	2	1
0	8	8	8	5	1	1	1	1

Solution

Use difference in gray-level values between pixels.

If the pixel gray-level values are close (within 2.5) and the pixels are neighbors draw an arc between the pixel and its 2-closest neighbors. Use 8-connected for neighbors.

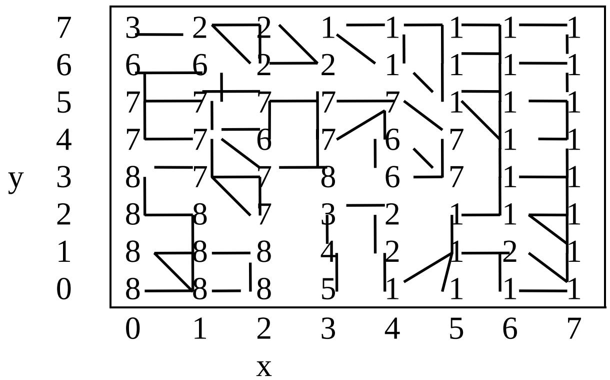

Image Data

1.3 Exercise

For the given data

a. Consider the pixels as nodes on a graph. Compute the property difference in gray levels as being the difference in gray-levels. Draw an arc between two nodes if the property is less than 2.5. Draw the graph for the given data for the lines y=0,1,2.

b. Determine the different regions from the graph.

c. If the gray-level 5 at point (3,0) is changed to a 6, how does this affect the regions?

	7	3	2	2	1	1	1	1	1
	6	6	6	2	2	1	1	1	1
	5	7	7	7	7	7	1	1	1
	4	7	7	6	7	6	7	1	1
y	3	8	7	7	8	6	7	1	1
	2	8	8	7	3	2	1	1	1
	1	8	8	8	4	2	1	2	1
	0	8	8	8	5	1	1	1	1
		0	1	2	3	4	5	6	7
				x					

Image Data

Solution

Use difference in gray-level values between pixels.

If the pixel gray-level values are close (within 2.5) and the pixels are neighbors draw an arc between the pixel and its neighbor. Use 8-connected for neighbors.

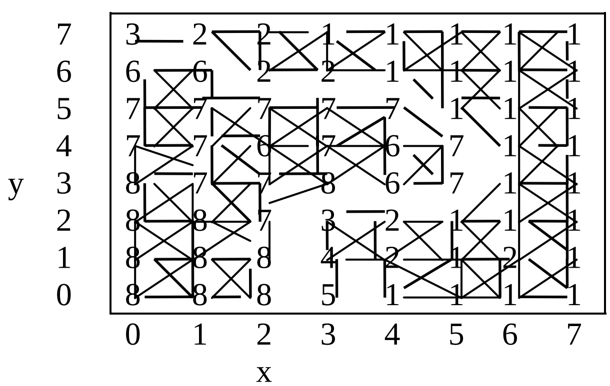

Image Data

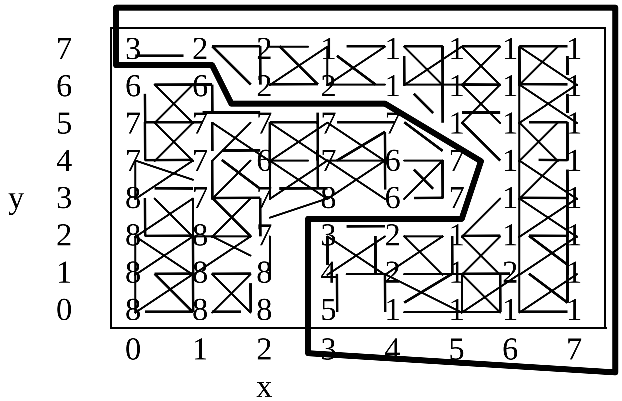

Image Data

region leakage refers to a region growing into another region through a narrow
connection
where the change in gray-levels is gradual but steady over the path.
the above regions do not have region leakage. it comes close to occurring on the path
that
transitions from gray level 1 to 2 to 3 to 4 to 5 but it stops. If the 5 value were changed to
a 6, then both regions would merge to form one region.

1.4 Seeded Region Growing

a. Describe the seeded region growing method

b. For the given data with the atomic regions marked, go through the calculations to show the next step in the process. Use formulas for efficient calculations.

Image Data

y \ x	0	1	2	3	4	5	6	7
7	3	2	2	1	1	1	1	1
6	6	5	2	2	1	1	1	1
5	7	7	7	7	7	1	1	1
4	7	7	6	7	6	7	1	1
3	8	7	7	8	6	7	1	1
2	8	8	7	3	2	1	1	1
1	8	8	8	4	2	1	2	1
0	8	8	8	5	1	1	1	1

b labeled region

a labeled region

Solution.

List T

points	(3,0)	(3,1)	(3,2)	(3,3)	(4,3)	(5,3)	(3,4)	(4,4)	(5,4)	(3,5)	(4,5)
δ	4	2	1	1	5	6	0	5	6	0	6

points	(1,6)	(2,6)	(3,6)	(0,7)	(1.7)	(2.7)
δ	2	1	1	4	5	1

Sorted list SSL

points	(3,4)	(3,5)	(3,2)	(3,3)	(2,6)	(3,6)	(1,7)	(3,1)	(1,6)	(3,0)	(0,7)
δ	0	0	1	1	1	1	1	2	2	4	4

points	(4,3)	(4,4)	(1,7)	(5,4)	(5,4)	(4,5)
δ	5	5	5	6	6	6

point (3,4) goes into the region labeled a

1.5 Facet region formation

a. Given the polynomial φ= a0 + a1*x + a2*y, and the masks for a0,a1, and a2, find the polynomial functions which fit the data at pixels (x,y)=(3,2) and (4,2).
b. Determine if (3,2) and (4,2) are part of the same surface using the concepts from facet region growing. Assume the best fitting polynomial function is the one centered on each pixel. Explain your analysis and conclusions fully.

Image Data

y \ x	0	1	2	3	4	5	6	7
7	3	2	2	1	1	1	1	1
6	6	5	2	2	1	1	1	1
5	7	7	7	6	2	1	1	1
4	7	7	6	7	2	1	1	1
3	8	7	7	6	2	1	1	1
2	8	8	7	6	2	1	1	1
1	8	8	8	8	2	1	2	1
0	8	8	8	8	1	1	1	1

a0 = 1/9 ×

1	1	1
1	(1)	1
1	1	1

a1 = 1/6 ×

-1	0	1
-1	(0)	1
-1	0	1

a2 = 1/6 ×

1	1	1
0	(0)	0
-1	-1	-1

Masks for a0, a1, and a2, the origin of the mask is at the center.

Solution

at point(3,2)

7	6	2
7	6	2
8	8	2

a0=5.33, a1=-2.67, a2= -.5

$\varphi(x, y) = 5.33 - 2.67x - .5y$

at point(4,2)

6	2	1
6	2	1
8	2	1

a0=3.22, a1=-2.83, a2= -.33

$\varphi(x, y) = 3.22 - 2.83x - .33y$

compare slopes
x-dir
-2.67, -2.83
y-dir

-.5, -.33
compare heights at midpoint
(3.5,2)
from polynomial at (3,2)
$\varphi(x, y) = 5.33 - 2.67x - .5y$ =5.33-2.67(.5)-.5(0)=4
from polynomial at (4,2)
$\varphi(x, y) = 3.22 - 2.83x - .33y$ =3.22-2.83(-.5)-.33(0)=3.22-1.41-0=4.63
The points are part of the same surface

1.6 Facet Region Formation

Explain this method for region formation. Explain each step in the method in a concise manner.

1.7 Exercise

Facet Region Formation

a. Given the polynomial p= a0 + a1*x + a2*y, and the masks for a0,a1, and a2, find the polynomial functions which fit the data at pixels (x,y)=(2,2) and (4,4).

b. Determine if (2,2) and (4,4) are part of the same surface. Explain your analysis and conclusions fully.

Image Data

y \ x	0	1	2	3	4	5	6	7
7	3	2	2	1	1	1	1	1
6	5	3	2	2	1	1	1	1
5	7	6	3	3	2	1	1	1
4	7	7	5	4	3	1	1	1
3	8	7	5	4	3	1	1	1
2	8	8	5	3	2	1	1	1
1	8	8	6	4	2	1	2	1
0	8	8	8	5	1	1	1	1

a0 (1/9):

1	1	1
1	(1)	1
1	1	1

a1 (1/6):

-1	0	1
-1	(0)	1
-1	0	1

a2 (1/6):

1	1	1
0	(0)	0
-1	-1	-1

Masks for a0, a1, and a2, the origin of the mask is at the center.

Solution

at point (2,2)
a0=(1/9)(7+5+4+8+5+3+8+6+4) = 5.55
a1=(1/6)(4 + 4+3 –7-8-8) = -2
a2=(1/6)(7+5+4-8-6-4) = -.33
φ(x,y)=5.55 - 2x-.33y
at point (4,4)
a0=(1/9)(3=2+1+4+3+1+4+3+1) = 2.44
a1=(1/6)(1 +1+1-3-4-4) = -1.33
a2=(1/6)(3+2+1-4-3-1) = -.33
φ(x,y)=2.44 - 1.33x - .33y

compare slopes
point (2,2)
a1=-2, a2=-.33

point(4,4)
a1=-1.33, a2=-.33
the slopes are all the same sign and are relatively close together

compare height at (3,3) half way point.
using equation from (2,2)
$(\Delta x,\Delta y)=(1,1)$
$\varphi(x,y)=\varphi(1,1)=5.55-2(1)-.33(1)= 3.22$
using equation from (4,4)
$(\Delta x,\Delta y)=(-1,-1)$
$\varphi(x,y)=\varphi(-1,-1)=2.44-1.33(-1)-.33(-1)=2.44+1.33+.33 = 4.1$

These two points could be considered to be on the same surface. It is a marginal call. They are both boundary type points between the two regions.

1.7.1

1.8 Region Formation and surfaces

a. Given the polynomial φ= a0 + a1*x + a2*y, and the masks for a0,a1, and a2, find the polynomial functions which fit the data at pixels (x,y)=(1,2) and (1,4).
b. Determine if (1,2) and (1,4) are part of the same surface using the concepts from facet region growing. Assume the best fitting polynomial function is the one centered on each pixel. Explain your analysis and conclusions fully.
c. Compute H and K the Mean and Gaussian curvature at pixels (1,2) and (1,4) using the fitted polynomials.
The general equations for H Gaussian and K mean curvature are given below.

$$H = \frac{(1+g_y^2)g_{xx} + (1+g_x^2)g_{yy} - 2g_x g_y g_{xy}}{2(1+g_x^2+g_y^2)^{3/2}}$$

$$K = \frac{g_{xx}g_{yy} - g_{xy}^2}{(1+g_x^2+g_y^2)^2}$$

d. What is the topographic labeling at each pixel. Does the labeling support the pixels being on the same surface? Explain.
e. Compute the surface normals at each pixel. Do they support the two pixels being on the same surface? Explain.

Image Data

y \ x	0	1	2	3	4	5	6	7
7	3	2	2	1	1	1	1	1
6	6	5	2	2	1	1	1	1
5	7	7	7	7	7	1	1	1
4	7	7	6	7	6	7	1	1
3	8	7	7	8	6	7	1	1
2	8	8	7	3	2	1	1	1
1	8	8	8	4	2	1	2	1
0	8	8	8	5	1	1	1	1

a0 = 1/9 ×

1	1	1
1	(1)	1
1	1	1

a1 = 1/6 ×

-1	0	1
-1	(0)	1
-1	0	1

a2 = 1/6 ×

1	1	1
0	(0)	0
-1	-1	-1

Masks for a0, a1, and a2, the origin of the mask is at the center.

Solution

Regions and surfaces.

at (1,2)

a_0 = 7.67 a_1 = -.33 a_2 = -33

$\varphi_1(x,y)$ = 7.67-.33x-.33y

at(1,4)
$a_0 = 7$ $a_1 = -.33$ $a_2 = -.16$ hence
$\varphi_2(x,y) = 7-.33x-.16y$

midpoint is (1,3)
height from $\varphi_1(x,y) = 7.67-.33x-.33y$ with x=0 and y=1 is 7.67-.33=7.34
height from $\varphi_2(x,y) = \varphi_2(x,y) = 7-.33x-.16y$
with x=0 and y=-1 is 7+.16=7.16

topographic label at (1,2)
H=0, K=0 the topographic label is flat

normals
at (1,2) the normal is

$$m1 = \frac{(-\varphi_x, -\varphi_y, 1)}{(1+\varphi_x^2+\varphi_y^2)^{\frac{1}{2}}} = \frac{(.33,.33,1)}{(1+.33^2+.33^2)^{\frac{1}{2}}} = (.3,.3,.91)$$

at (1,4) the normal is

$$m2 = \frac{(-\varphi_x, -\varphi_y, 1)}{(1+\varphi_x^2+\varphi_y^2)^{\frac{1}{2}}} = \frac{(.33,.16,1)}{(1+.33^2+.16^2)^{\frac{1}{2}}} = (.31,.15,.94)$$

<m1,m2>= .3*.31 +.15*.3+.94*.91 = .09+.045+.856=.99
the angles are very close, this supports the pixels being part of the same surface

1.9 Surface fitting and Regions.

At the pixels (2,4) and (2,3) compute the polynomials which fit the image data.
10 points. Compute H and K the Mean and Gaussian curvature at pixels (2,4) and (2,3) using the fitted polynomials.

The general equations for H Gaussian and K mean curvature are given below.

$$H = \frac{(1+g_y{}^2)g_{xx} + (1+g_x{}^2)g_{yy} - 2g_x g_y g_{xy}}{2(1+g_x{}^2+g_y{}^2)^{3/2}}$$

$$K = \frac{g_{xx}g_{yy} - g_{xy}{}^2}{(1+g_x{}^2+g_y{}^2)^2}$$

d. 5 points. What is the topographic labeling at each pixel. Does the labeling support the pixels being on the same surface? Explain.
e. 10 points. Compute the surface normals at each pixel. Use the polynomials to compute the normals. Do the normals support the two pixels being on the same surface? Explain.

$$m = \frac{(-g_x, -g_y, 1)}{(1+g_x^2+g_y^2)^{\frac{1}{2}}}$$ equation for surface normal.

Image Data

y \ x	0	1	2	3	4	5	6	7
7	3	2	2	1	1	1	1	1
6	6	5	2	2	1	1	1	1
5	7	7	7	6	2	1	1	1
4	7	7	6	7	2	1	1	1
3	8	7	7	6	2	1	1	1
2	8	8	7	6	2	1	1	1
1	8	8	8	8	2	1	2	1
0	8	8	8	8	1	1	1	1

a0 = 1/9 ×

1	1	1
1	(1)	1
1	1	1

a1 = 1/6 ×

-1	0	1
-1	(0)	1
-1	0	1

a2 = 1/6 ×

1	1	1
0	(0)	0
-1	-1	-1

Masks for a0, a1, and a2, the origin of the mask is at the center.
Solution
at (2,4)
a0=6.67, a1=-.33, a2=0;
$\varphi(x,y) = 6.67 - .33x$
H=K=0, flat

$$m1 = \frac{(.33,0,1)}{(1+.33^2)^{.5}}$$

at (2,3)
a0=6.78 a1=-.5, a2=-.167
$\varphi(x,y) = 6.78 - .5x - .167y$
H=K=0, flat

$$m2 = \frac{(.5,.167,1)}{(.5^2 + .167^2 + 1)^{.5}}$$

The topographic labels are both flat which supports a part of the same surface. The surface normals are both in the same directions which again supports the same surface since the surface is flat.