Exercises- Binary Morphology

1.1 Exercise

Give the definition for dilation $A \oplus B$ where A is the image and B is the structuring element. Give the definition for erosion $A \Theta B$.

solution

$A \oplus B = \{a + b | a \in A \text{ and } b \in B\}$

$A \Theta B = \{p | p + b \in A \text{ for every } b \text{ in } B\}$

1.2 Exercise

Give the definition for open $A \circ B$ and close $A \bullet B$

1.3 Exercise

Prove $A \subseteq A \bullet B$

solution

let a be in A then a+b is in $A \oplus B$ for every b . This implies that a is in $(A \oplus B) \Theta B = A \bullet B$.

1.4 Exercise

Prove $A \circ B \subseteq A$

Solution

$p \in A \circ B \Rightarrow p \in (A \Theta B) \oplus B$. This implies p=u+b' where $u \in (A \Theta B)$, *for some* $b' \in B$. From the definition of erosion $u \in (A \Theta B) \Rightarrow u + b \in A, \forall b \in B$. Let b be b' then p=u+b' is in A. Therefore $A \circ B \subseteq A$.

1.5 Exercise

Prove $A \Theta (B \oplus C) = (A \Theta B) \Theta C$.

What is the significance of this result?

Given B

1	1	1	
1	1	1	B
1	1	1	

then give a B1 and B2 where $B = B1 \oplus B2$

Solution

Let $p \in (A \Theta B) \Theta C) \Rightarrow p + b + c \in A, \forall b \in B, \forall c \in C$. Let $D = B \oplus C$, then $d = b + c \in D \; \forall b \in B, \forall c \in C$. This implies that $p + b + c \in A \Theta D = A \Theta (B \oplus C)$. Hence $(A \Theta B) \Theta C) \subseteq (A \Theta (B \oplus C))$.

Let $p \in A \Theta (B \oplus C) \Rightarrow p + d \in A, \forall d \in B \oplus C$. From the definition of dilation. This implies that $p + b + c \in A, \forall b \in B, \forall c \in C$. Therefore

$((p+b)+c \in A, \forall b \in B, \forall c \in C \Rightarrow (p+b) \in (A\Theta B)\ \forall b \in B$. And
$p+b+c \in \forall b, \forall c \in B \Rightarrow\ p \in (A\Theta B)\Theta B$. Hence $(A\Theta(B \oplus C) \subseteq (A\Theta B)\Theta C$

	1	
	1	
	1	

B1

1	1	1

B2

1.6 Exercise

Prove $A\oplus(B\cup C)=(A\oplus B)\cup(A\oplus C)$

1.7 Exercise

Prove $A\oplus(B\oplus C)=(A\oplus B)\oplus C$

1.8 Exercise

$(A\Theta B)^c = A^c\Theta\hat{B}$

where $\hat{B}$ is the reflection of set B
Give an interpretation of this result.

		1	1		
		1	1		

A

1	1	

B

1.9 Exercise

prove $(A\circ B)=(A^c \bullet B)^c$. What does this result say in words?

1.10 Exercise

$(A\bullet B)=(A^c\circ B)^c$

1.11 Exercise

Prove idempotent

$(A \circ B) \circ B = A \circ B$

$(A \bullet B) \bullet B = A \bullet B$

1.12 Exercise

a. Define binary erosion $A \Theta B$

Solution

$p \in A\Theta B \Rightarrow p + b \in A \; \forall b \in B$

b. Prove for A,B,C binary objects $A \oplus (B \oplus C) = (A \oplus B) \oplus C$ and $A\Theta(B \oplus C) = (A\Theta B)\Theta C$

Solution

let $p \in A\Theta(B \oplus C) \Rightarrow p + b + c \in A \quad \forall b \in B \quad \forall c \in C$

$\Rightarrow p + b \in A\Theta B \quad \forall b \in B \quad \Rightarrow p + b + c \in (A\Theta B)\Theta C$

hence $A\Theta(B \oplus C) \subseteq (A\Theta B)\Theta C)$

Let $p \in (A\Theta B)\Theta C) \Rightarrow p + c \in (A\Theta B) \quad \forall c \in C$

$(p + c) \in (A\Theta B) \quad \forall c \in C \Rightarrow (p + c) + b \in A \quad \forall b \in B \Rightarrow p + b + c \in A \quad \forall b \in B \quad \forall c \in C$

$\Rightarrow p \in A\Theta(B \oplus C) \Rightarrow (A\Theta B)\Theta C) \subseteq A\Theta(B \oplus C)$

therefore we have equality

1.13 Exercise

Explain the noise removing properties of the open operator

Explain the noise removing properties of the close operator.

What noise points will be removed by open in the following example?

y									A
7	0	0	0	1	0	0	0	0	
6	0	0	0	1	0	0	0	0	
5	0	0	0	1	0	0	0	0	
4	0	0	1	0	0	0	1	1	
3	0	0	1	0	0	0	1	0	
2	0	0	0	0	0	0	0	1	
1	0	0	1	0	0	1	0	0	
0	0	0	0	0	0	0	0	0	
	0	1	2	3	4	5	6	7	
		x							

	B
1	
1	

1.14 Exercise

Define the boundary location operator.
Apply it to the following data set.

	7	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0
	5	0	0	1	1	1	0	0	0
	4	0	0	1	1	1	0	0	0
y	3	0	0	1	1	1	0	0	0
	2	0	0	1	1	1	0	0	0
	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7
			x						

1	1	1
1	1	1
1	1	1

solution

$(A \oplus B) - (A \Theta B)$

7		0	0	0	0	0	0	0	0
	6	0	1	1	1	1	1	0	0
	5	0	1	1	1	1	1	0	0
	4	0	1	1	1	1	1	0	0
y	3	0	1	1	1	1	1	0	0
	2	0	1	1	1	1	1	0	0
	1	0	1	1	1	1	1	0	0
	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7
			x						

7		0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0

4	0	0	0	1	0	0	0	0
y 3	0	0	0	1	0	0	0	0
2	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
	0	1	2	3	4	5	6	7
		x						

7	0	0	0	0	0	0	0	0
6	0	1	1	1	1	1	0	0
5	0	1	1	1	1	1	0	0
4	0	1	1	0	1	1	0	0
y 3	0	1	1	0	1	1	0	0
2	0	1	1	1	1	1	0	0
1	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0
	0	1	2	3	4	5	6	7
		x						

1.15 Exercise

14. define conditional dilation of set A by set B in set C.
explain how this can be applied to region filling.
do it for the following data where the pixels inside the 1's are to be filled.

7	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
5	0	0	1	1	1	1	1	0
4	0	0	1	0	0	0	1	0
y 3	0	0	1	0	0	0	1	0
2	0	0	1	1	0	0	1	0
1	0	0	0	1	1	1	1	0
0	0	0	0	0	0	0	0	0
	0	1	2	3	4	5	6	7
		x						

Solution

1	1	1
1	1	1
1	1	1

$$(A \oplus B)_C = (A \oplus B) \cap C$$

The set C is the complement of the set of 1's in the original data

7	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
4	0	0	0	1	1	1	0	0
y 3	0	0	0	1	1	1	0	0
2	0	0	0	0	1	1	0	0
1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
	0	1	2	3	4	5	6	7
		x						

1.16 Exercise

a. For the given data compute the dilate, erode, open, close operations. The object is denoted by the 1's in the image

	1	1	1						
	1	1	1		1		1		
	1	1	1		1	1	1	1	
		1			1	1	1		
	1	1	1	1	1	1	1		
	1	1	1						
	1	1	1						

image

1	1	1
1	1	1
1	1	1

B

Solution:

d	d	d	d	d					
d	1	1	1	d	d	d	d	d	
d	1	1	1	d	1	d	1	d	d
d	1	1	1	d	1	1	1	1	d
d	d	1	d	d	1	1	1	d	d
d	1	1	1	1	1	1	1	d	
d	1	1	1	d	d	d	d	d	
d	1	1	1	d					
d	d	d	d	d					

dilate

	1	1	1						
	1	1e	1		1		1		
	1	1	1		1	1	1	1	
		1			1	1e	1		
	1	1	1	1	1	1	1		
	1	1e	1						
	1	1	1						

Erode

	1	1	1						
	1	e	1						
	1	1	1		1	1	1		
					1	e	1		
	1	1	1		1	1	1		
	1	e	1						
	1	1	1						

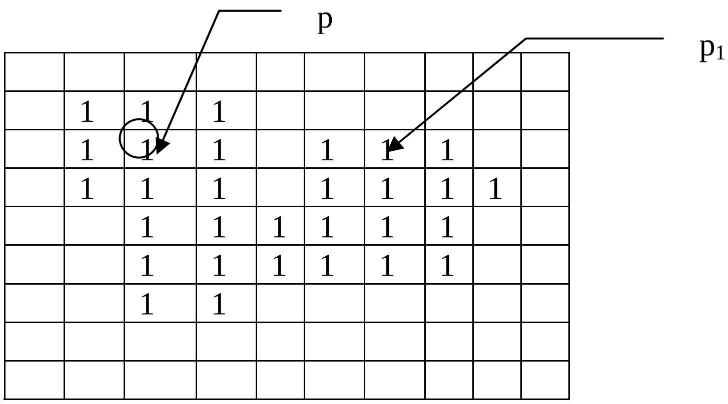
open

	1	1	1						
	1	1	1	c	1	c	1		
	1	1	1	c	1	1	1	1	
	c	1	c	c	1	1	1		
	1	1	1	1	1	1	1		
	1	1	1						
	1	1	1						

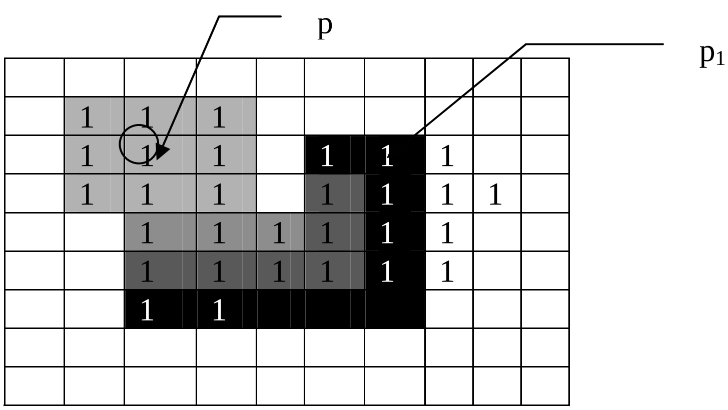
close

b. Compute 2B, 3B, 4B

c. Starting with the circled point p
 compute $p \oplus B, (p \oplus B) \oplus B, \ldots, (p \oplus B) \ldots \oplus B$ four times, conditioned, intersected ,with the set defined by the 1's. Do this iteratively not with 3B, 4B etc. What is the geodetic distance between p and p_1?



Solution



the geodesic distance is 4.

1.17 Exercise

	1	1	1						
	1	1	1		1		1		
	1	1	1		1	1	1	1	
		1			1	1	1		
	1	1	1	1	1	1	1		
	1	1	1						
	1	1	1						

image

1	1	1
1	1	1
1	1	1

B

a.

Define and compute 2B, 3B

Solution

$2B = B \oplus B$

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

2B

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

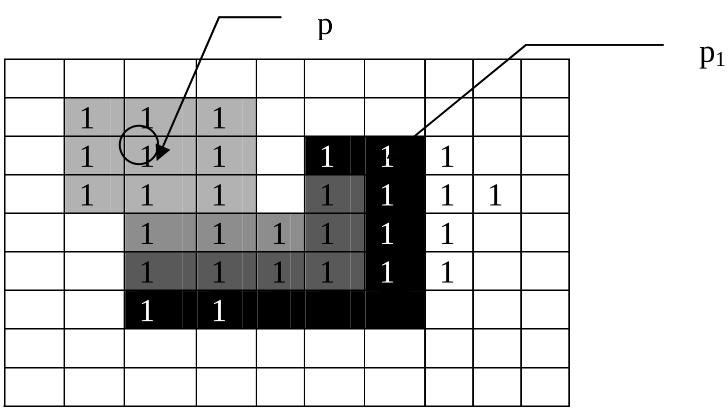
3B

b.
Starting with the circled point p,
compute $p \oplus B, (p \oplus B) \oplus B, \ldots, (p \oplus B) \ldots \oplus B$ four times, conditioned, intersected, with the set defined by the 1's. Do this iteratively not with 2B, 3B etc. What is the geodetic distance between p and p_1?

p p_1

	1	1	1						
	1	1	1		1	1	1		
	1	1	1		1	1	1	1	
		1	1	1	1	1	1		
		1	1	1	1	1	1		
		1	1						

Solution



the geodesic distance is 4.

1.18 Exercise

Give a circular structuring element for a 7 by 7 grid with the origin at the center. The radius of the circle should be 3.

Solution

0	0	0	1	0	0	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
1	1	1	1	1	1	1
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	0	0	1	0	0	0

1.19 Exercise

Define the hit-or-miss transform

Apply to recognize the rectangle in the following data.

	7	0	0	0	1	0	0	0	0
	6	0	0	0	1	0	0	1	0
	5	0	0	0	0	0	0	0	0
	4	0	1	1	0	0	1	1	1
y	3	0	1	1	0	0	1	1	0
	2	0	1	1	0	0	1	1	0
	1	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7
			x						

Solution

$(A\Theta B_1)\cap(A^c\Theta B_2)$

	1	1	
	1	1	
	1	1	

	1	1	1	1	
	1			1	
	1			1	
	1			1	
	1	1	1	1	

	7	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0
y	3	0	1	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7
			x						

1.20 Exercise

a. Define the hit-and-miss operator. What is it commonly used for?

Solution

A⊛ B = $(A \Theta B_1) \cap (A^c \Theta B_2)$

b. Give structuring elements to use the hit-or-miss operator to detect the following object.

	1		
1	1	1	
1	1	1	
	1		

Solution

					1	1	1	1	1	
	1				1	1	0	1	1	
1	1	1			1	0	0	0	1	
1	1	1			1	0	0	0	1	
	1				1	1	0	1	1	
					1	1	1	1	1	

B_1^1 B_1^2

c.

For the following image apply the hit-and-miss operator from part b and give the results.

		1				1	1				
	1	1	1		1	1	1				
	1	1	1		1	1	1				
		1				1					

Solution

		1									

d. Define hit-and-miss templates to pass both of the above objects but not to pass others. explain why it works.

Solution

					1	1	1	1	1	
	1				1	1	0	0	1	
1	1	1			1	0	0	0	1	
1	1	1			1	0	0	0	1	
	1				1	1	0	1	1	
					1	1	1	1	1	

B_1^1 B_1^2

1.21 Exercise

a. Give the open of the given data with the Bv structuring element.
Explain how you obtained your results.

b.
Give the open of the given data with the Bcu structuring element. Explain how you obtained your results. Interpret your results.

c.
Give the open of the given data with the Bcd structuring element. Explain how you obtained your results. Interpret your results.

	y								
	7	0	0	0	1	0	0	0	0
	6	0	0	0	1	0	0	0	0
	5	0	0	0	1	0	0	0	0
	4	0	0	1	1	0	0	1	1
y	3	0	1	1	0	0	0	1	0
	2	0	0	0	0	0	0	0	1
	1	0	0	1	0	0	1	0	0
	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7
			x						

0	1	0	
1	1	0	Bcu
0	0	0	

0	1	0	
0	1	0	Bv
0	1	0	

0	0	0	
0	1	1	Bcd
0	1	0	

solution

	7	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0
	5	0	0	0	1	0	0	0	0
	4	0	0	1	1	0	0	0	0
y	3	0	1	1	0	0	0	0	0
	2	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7
			x						

	7	0	0	0	1	0	0	0	0
	6	0	0	0	1	0	0	0	0
	5	0	0	0	1	0	0	0	0
	4	0	0	0	1	0	0	0	0
y	3	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7
			x						

	7	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0
	4	0	0	1	1	0	0	1	1
y	3	0	0	1	0	0	0	1	0
	2	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7
			x						

1.22 Exercise

a. Give the open of the given data with the B4 structuring element.
Explain how you obtained your results.

b.
Give the open of the given data with the Bh structuring element. Explain how you obtained your results. Interpret your results.

c.
Give the open of the given data with the Bd structuring element. Explain how you obtained your results. Interpret your results.

y		0	1	2	3	4	5	6	7
	7	0	0	0	1	0	0	0	0
	6	0	0	0	1	1	0	0	0
	5	0	0	1	1	1	0	0	0
	4	0	0	0	1	0	1	1	1
y	3	0	1	1	0	0	0	1	0
	2	0	0	0	0	0	0	0	1
	1	0	0	1	0	0	1	0	0
	0	0	0	0	0	0	0	0	0
	x	0	1	2	3	4	5	6	7

0	1	0	
1	1	1	B4
0	1	0	

0	0	0	
1	1	1	Bh
0	0	0	

1	0	0	
0	1	0	Bd
0	0	1	

Solution

	7	0	0	0	0	0	0	0	0
	6	0	0	0	1	0	0	0	0
	5	0	0	1	1	1	0	0	0
	4	0	0	0	1	0	0	0	0
y	3	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7
			x						

open with B4

	7	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0
	5	0	0	1	1	1	0	0	0
	4	0	0	0	0	0	1	1	1
y	3	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7
			x						

open with Bh

1.23 Exercise

For binary images, with S the image and B the structuring element given.

a.

Give B, 2B, 3B

b.

define the pattern spectra. p(k). Give all relevant equations. Explain your work clearly.

c.

what features of S does p(k) represent?

d.

compute p(k) for the given data and interpret your results.

S

7	0	0	1	1	1	0	0	0
	0	0	0	1	1	1	0	0
	0	0	1	1	1	1	1	0
	0	0	0	1	1	1	1	1
	0	0	1	0	1	0	0	0
	0	1	1	1	0	0	0	0
	0	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0
	0							7

x

B

y			
		1	
	1	1	1
		1	

x

Solution

7	0	0	0	0	1	0	0	0
	0	0	0	1	1	1	0	0
	0	0	1	1	1	1	1	0
	0	0	0	1	1	1	0	0
	0	0	1	0	1	0	0	0
y	0	1	1	1	0	0	0	0
	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0
	0							7
				x				

Open with B

		1		
	1	1	1	
1	1	1	1	1
	1	1	1	
		1		

2B

7	0	0	0	0	1	0	0	0
	0	0	0	1	1	1	0	0
	0	0	1	1	1	1	1	0
	0	0	0	1	1	1	0	0
	0	0	0	0	1	0	0	0
y	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
	0							7
				x				

Open with 2B

			1			
		1	1	1		
	1	1	1	1	1	
1	1	1	1	1	1	1
	1	1	1	1	1	
		1	1	1		
			1			

3B

A(0) = 23
A(1) = 18
A(2) = 13
A(3) = 0

$$p(k) = \frac{A(k) - A(k+1)}{A(0)}$$

p(0)=5/23 , area of elements of size <B
p(1)=5/23, area of elements of size B
p(2)=13/23, area of elements of size 2B
p(3)=0, area of elements of size 3B

1.24 Exercise

For binary images, with S the image and B the structuring element given.

a.

Give B, 2B, 3B

b.

define the pattern spectra. p(k). Give all relevant equations. Explain your work clearly.

c.

what features of S does p(k) represent?

d.

compute p(k) for the given data and interpret your results.

7	0	1	1	1	1	1	0	0	
	0	1	1	1	1	1	0	0	
	0	1	1	1	1	1	1	0	
y	0	1	1	1	1	1	1	1	S
	0	1	1	1	1	1	0	0	
	0	1	1	1	0	0	0	0	
	0	1	1	1	0	0	0	0	
0	0	1	1	1	0	0	0	0	
	0							7	
		x							

y	1	1	1	
	1	1	1	B
	1	1	1	
		x		

Solution

	7	0	1	1	1	1	1	0	0
		0	1	1	1	1	1	0	0
		0	1	1	1	1	1	0	0
y		0	1	1	1	1	1	0	0
		0	1	1	1	1	1	0	0
		0	1	1	1	0	0	0	0
		0	1	1	1	0	0	0	0
	0	0	1	1	1	0	0	0	0
		0							7
				x					

S

Open with B

1	1	1	1	1
	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

2B

	7	0	1	1	1	1	1	0	0
		0	1	1	1	1	1	0	0
		0	1	1	1	1	1	0	0
y		0	1	1	1	1	1	0	0
		0	1	1	1	1	1	0	0
		0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0
		0							7
				x					

S

Open with 2B

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

3B

A(0) = 37
A(1) = 34
A(2) = 25

A(3) = 0

$$p(k) = \frac{A(k) - A(k+1)}{A(0)}$$

p(0)=3/37 , area of elements of size <B
p(1)=9/37, area of elements of size B
p(2)=25/37, area of elements of size 2B
p(3)=0, area of elements of size 3B

1.25 Exercise

For binary images, with S the image and B the structuring element given.

a.

Define the pattern spectra. p(k). Give all relevant equations. Explain your work clearly.

Solution

$$p(k) = \frac{A(k) - A(k+1)}{A(0)}$$

A(k) is the area of $S \circ kB$.

b.

What features of S does p(k) represent?

Solution

p(0) area of elements of size <B
p(1) area of elements of size B
p(2) area of elements of size 2B
p(3) area of elements of size 3B

c.

Compute the pattern spectrum p(k) for the given data and interpret your results.

7	0	0	1	1	1	0	0	0	
	0	0	0	1	1	1	0	0	
	0	0	1	1	1	1	1	0	
	0	0	0	1	1	1	1	1	S
	0	0	1	0	1	0	0	0	
	0	1	1	1	0	0	0	0	
	0	1	1	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	
	0							7	

x

		1		
y	1	1	1	B
		1		

x

Solution

7	0	0	0	0	1	0	0	0
	0	0	0	1	1	1	0	0
	0	0	1	1	1	1	1	0
	0	0	0	1	1	1	0	0
	0	0	1	0	1	0	0	0
y	0	1	1	1	0	0	0	0
	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0
	0							7
				x				

Open with B

		1		
	1	1	1	
1	1	1	1	1
	1	1	1	
		1		

2B

7	0	0	0	0	1	0	0	0
	0	0	0	1	1	1	0	0
	0	0	1	1	1	1	1	0
	0	0	0	1	1	1	0	0
	0	0	0	0	1	0	0	0
y	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
	0							7
				x				

Open with 2B

			1			
		1	1	1		
	1	1	1	1	1	
1	1	1	1	1	1	1
	1	1	1	1	1	
		1	1	1		
			1			

3B

A(0) = 23
A(1) = 18
A(2) = 13
A(3) = 0

$$p(k) = \frac{A(k) - A(k+1)}{A(0)}$$

p(0)=5/23 , area of elements of size <B
p(1)=5/23, area of elements of size B
p(2)=13/23, area of elements of size 2B
p(3)=0, area of elements of size 3B