# 1. Exercises for Gray-Scale Morphology

## 1.1 Exercise

a. Give the definition for gray- level erosion, $g\Theta f$ , where g is the image and f is the structuring element.

b. Give the definition for gray-level dilation $g \oplus f$ .

solution

$g\Theta f = \min\{g(p+u) - f(u) | u \in Dom[f],\ p+u \in Dom[g]\}$

$g \oplus f = \max\{g(p-u) + f(u) | u \in Dom[f]\}$

## 1.2 Exercise

a. 6 points

give the definition for gray- scale erosion, $g\Theta f$, where g is the image and f is the structuring element.

**Solution.**

$g\Theta f = \min\{\ g(p+u) - f(u) |\ p+u \in Dom[g], u \in Dom[f]\ \}$

b. 6 points

Give the definition for gray-scale dilation $g \oplus f$.

**Solution**

$g \oplus f = \max\{g(p-u) + f(u) \mid u \in Dom[f]\ \}$

c. 6 points

if f is zero on its domain, what is the equation for gray-scale erosion and dilation.

**Solution**

$g\Theta f = \min\{\ g(p+u) |\ p+u \in Dom[g], u \in Dom[f]\ \}$

$g \oplus f = \max\{g(p-u) \mid u \in Dom[f]\ \}$

d. 7 points

compute the gray-scale erosion of the given data.

e. 8 points

define gray-scale open. What features of the image g will it remove?

**Solution**

$g \circ f = (g\Theta f) \oplus f$

f. 12 points

compute the open of the given data. Interpret your results. What features are preserved and what features are removed?

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 5 | 1 | 1 | 9 | 1 | 1 | 1 |
| | 1 | 5 | 1 | 1 | 9 | 9 | 1 | 1 |
| y | 1 | 1 | 1 | 1 | 9 | 9 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 0 | | | | | | | 7 |

x

g

| | | | |
|---|---|---|---|
| | 0 | 0 | 0 |
| y | 0 | ⓪ | 0 |
| | 0 | 0 | 0 |

x

f

**Solution**

y

g

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | | | | | | | | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | |
| 0 | | | | | | | | |
| | 0 | | | | | | | 7 |

x

erode

y

g

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 0 | | | | | | | 7 |

x

open

the open operator removes all the small peaks in the image

## 1.3 Exercise

a. If f is zero on its domain, what are the equations for gray-level erosion and dilation.

Solution

$$g\Theta f = \min\{\ g(p+u) |\ p+u \in Dom[g], u \in Dom[f]\ \}$$

$$g \oplus f\ = \max\{\ g(p-u) | u \in Dom[f]\ \}$$

b. Compute the gray-level erosion of the given data.

c. Compute the gray-level dilation of the given data.

d. Define gray-level open. What features of the image g will it remove?

**Solution**

$$g \circ f = (g\Theta f) \oplus f$$

e. Compute the open of the given data.

f. Define the gray-level close. what features of the image will it remove?

g. Compute the close of the given data.

|  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
| 7 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
|  | 6 | 6 | 7 | 6 | 6 | 6 | 6 | 6 |
|  | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| y | 6 | 6 | 6 | 1 | 6 | 6 | 6 | 6 |
|  | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
|  | 6 | 6 | 6 | 6 | 9 | 9 | 6 | 6 |
|  | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 0 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
|  | 0 |  |  |  |  |  |  | 7 |

x

g

|  |  |  |  |
|---|---|---|---|
|  | 0 | 0 | 0 |
| y | 0 | 0 | 0 |
|  | 0 | 0 | 0 |

x

f

**Solution**

y

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | | | | | | | | |
| | | 6 | 6 | 6 | 6 | 6 | 6 | |
| | | 6 | 1 | 1 | 1 | 6 | 6 | |
| | | 6 | 1 | 1 | 1 | 6 | 6 | |
| | | 6 | 1 | 1 | 1 | 6 | 6 | |
| | | 6 | 6 | 6 | 6 | 6 | 6 | |
| | | 6 | 6 | 6 | 6 | 6 | 6 | |
| 0 | | | | | | | | |
| | 0 | | | | | | | 7 |

g

x

erode

y

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| | 6 | 6 | 6 | 1 | 6 | 6 | 6 | 6 |
| | 6 | 6 | 9 | 6 | 6 | 6 | 6 | 6 |
| | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 0 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| | 0 | | | | | | | 7 |

g

x

open

### 1.4 Exercise

a. Define the top-hat operator.

b.What type features of the image would it locate?

c. Compute the top-hat command for the given data and interpret the results.

y

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| | 6 | 6 | 7 | 6 | 6 | 6 | 6 | 6 |
| | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| | 6 | 6 | 6 | 1 | 6 | 6 | 6 | 6 |
| | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| | 6 | 6 | 6 | 6 | 9 | 9 | 6 | 6 |
| | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 0 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| | 0 | | | | | | | 7 |

x

g

y

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | ⓪ | 0 |
| 0 | 0 | 0 |

x

f

data

**Solution**

y

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 3 | 3 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | | | | | | | 7 |

x

g

tophat

## 1.5 Exercise

a. Define the top-hat operator for gray-level morphology
What type features will it locate?

b Define the boundary operator for gray-level morphology.

**Solution.**

if g is the image function and f is the structuring function
then $grad(g) = (g \oplus f) - (g \Theta f)$.

$tophat(g) = g - (g \circ f)$

the $(g \circ f)$ open is beneath g. Hence the tophat operator is useful for finding objects that are peaks on a flat background.

**c.**

For the given data compute
the dilation of g with f
the erosion of g with f
the gradient of g with f
the open of g with f
the top-hat of g with f

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 2 | 1 | 1 | 2 | 1 | 1 |
| 1 | 2 | 1 | 1 | 2 | 2 | 2 | 1 |
| 1 | 2 | 2 | 8 | 7 | 2 | 2 | 1 |
| 1 | 1 | 1 | 9 | 8 | 1 | 1 | 1 |
| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

g

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

f

**Solution**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 |
| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| 1 | 2 | 2 | 8 | 8 | 8 | 7 | 2 | 2 | 1 |
| 1 | 2 | 2 | 9 | 9 | 9 | 8 | 2 | 2 | 1 |
| 1 | 2 | 2 | 9 | 9 | 9 | 8 | 2 | 2 | 1 |
| 1 | 2 | 2 | 9 | 9 | 9 | 8 | 2 | 1 | 1 |
| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

g dilate f

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

g erode f

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 7 | 7 | 7 | 6 | 2 | 1 |
| 1 | 1 | 8 | 8 | 8 | 7 | 2 | 1 |
| 1 | 1 | 8 | 8 | 8 | 7 | 1 | 1 |
| 1 | 1 | 8 | 8 | 8 | 7 | 2 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

grad of g

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

g open f

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 7 | 6 | 1 | 1 | 0 |
| 0 | 0 | 0 | 8 | 7 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

tophat

## 1.6 Exercise

**a.**

give the definition for gray- scale erosion, $g\Theta f$ , where g is the image and f is the structuring element.

**Solution.**

$$g\Theta f = \min\left\{\ g(p+u)-f(u)|\ p+u \in Dom[g], u \in Dom[f]\ \right\}$$

**b.**

Give the definition for gray-scale dilation $g \oplus f$ .

**Solution**

$$g \oplus f = \max\left\{g(p-u)+f(u)\ |\ u \in Dom[f]\ \right\}$$

**c.**

if f is zero on its domain, what is the equation for gray-scale erosion and dilation.

**Solution**

$$g\Theta f = \min\left\{\ g(p+u)|\ p+u \in Dom[g], u \in Dom[f]\ \right\}$$

$$g \oplus f = \max\left\{g(p-u)\ |\ u \in Dom[f]\ \right\}$$

**d.**

compute the gray-scale erosion of the given data.

**e.**

compute the gray-scale dilation of the given data

**f.**

define gray-scale open. What features of the image g will it remove?

**Solution**

$$g \circ f = (g\Theta f) \oplus f$$

**g.**

compute the open of the given data. Interpret your results. What features are preserved and what features are removed?

**h.**

Define the top-hat operator for gray-level morphology

What type features will it locate?

**i.**

Define the boundary operator for gray-level morphology.

**Solution.**

if g is the image function and f is the structuring function

then $grad(g) = (g \oplus f) - (g\Theta f)$.

**j.**

For the given data compute

the top-hat of g with f

**k.**

For the given data compute

the gradient of g with f

y

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 2 | 1 | 1 | 9 | 1 | 1 | 1 |
| | 1 | 2 | 1 | 1 | 9 | 9 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 9 | 9 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 9 | 9 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

g

x

y

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | ⓪ | 0 |
| 0 | 0 | 0 |

f

x

**Solution**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | | | | | | | | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | |
| y | | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | |
| 0 | | | | | | | | |
| | 0 | | | | | | | 7 |

x

g

Erode

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 2 | 2 | 9 | 9 | 9 | 1 | 1 | 1 |
| 1 | 2 | 2 | 2 | 9 | 9 | 9 | 9 | 1 | 1 |
| 1 | 2 | 2 | 2 | 9 | 9 | 9 | 9 | 1 | 1 |
| 1 | 2 | 2 | 2 | 9 | 9 | 9 | 9 | 1 | 1 |
| 1 | 1 | 1 | 1 | 9 | 9 | 9 | 9 | 1 | 1 |
| 1 | 1 | 1 | 1 | 9 | 9 | 9 | 9 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |

x

7

g

0

y

dilate

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| y | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 0 | | | | | | | 7 |

x

g

open

the open operator removes all the small peaks in the image

**h.**

Define the top-hat operator for gray-level morphology

What type features will it locate?

**Solution.**

$tophat(g) = g - (g \circ f)$

the $(g \circ f)$ open is beneath g. Hence the tophat operator is useful for finding objects that are peaks on a flat background.

**i.**

Define the boundary operator for gray-level morphology.

**Solution.**

if g is the image function and f is the structuring function

then $grad(g) = (g \oplus f) - (g \Theta f)$.

**j**.

For the given data compute

the top-hat of g with f

**k.**

For the given data compute

the gradient of g with f

|  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | 0 | 1 | 0 | 0 | 8 | 0 | 0 | 0 |
| y | 0 | 1 | 0 | 0 | 8 | 8 | 0 | 0 |
|  | 0 | 0 | 0 | 0 | 8 | 8 | 0 | 0 |
|  | 0 | 0 | 0 | 0 | 8 | 8 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|  |  | x |  |  |  |  |  |  |

Tophat operation

|  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
| 7 |  |  |  |  |  |  |  |  |
|  |  | 1 | 1 | 8 | 8 | 8 | 0 |  |
|  |  | 1 | 1 | 8 | 8 | 8 | 8 |  |
|  |  | 1 | 1 | 8 | 8 | 8 | 8 |  |
| y |  | 1 | 1 | 8 | 8 | 8 | 8 |  |
|  |  | 0 | 0 | 8 | 8 | 8 | 8 |  |
|  |  | 0 | 0 | 8 | 8 | 8 | 8 |  |
| 0 |  |  |  |  |  |  |  |  |
|  | 0 |  |  |  |  |  |  | 7 |
|  |  | x |  |  |  |  |  |  |

Boundary operation

## 1.7 Exercise

y

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 1 | 2 | 1 | 1 | 9 | 1 | 1 | 1 |
| | 1 | 2 | 1 | 1 | 9 | 9 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 9 | 9 | 1 | 1 |
| | 1 | 1 | 1 | 1 | 9 | 9 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

x

g

y

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

f

x

**a.**

Give the definition for gray- scale erosion, $g\Theta f$, where g is the image and f is the structuring element.

**Solution.**

$$g\Theta f = \min\{\ g(p+u) - f(u) |\ p+u \in Dom[g], u \in Dom[f]\ \}$$

**b.**

Give the definition for gray-scale dilation $g \oplus f$.

**Solution**

$$g \oplus f = \max\{g(p-u) + f(u) \mid u \in Dom[f]\ \}$$

**c.**

If f is zero on its domain, what is the equation for gray-scale erosion and dilation?

**Solution**

$$g\Theta f = \min\{\ g(p+u) |\ p+u \in Dom[g], u \in Dom[f]\ \}$$

$$g \oplus f = \max\{g(p-u) \mid u \in Dom[f]\ \}$$

**d**.

Compute the gray-scale erosion of the given data.

**Solution**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 7 | | | | | | | | | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | | g |
| y | | 1 | 1 | 1 | 1 | 1 | 1 | | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | | |
| | | 1 | 1 | 1 | 1 | 1 | 1 | | |
| 0 | | | | | | | | | |
| | 0 | | | | | | | 7 | |
| | | x | | | | | | | |

Erode

**e.**

Define gray-scale open. What features of the image g will it remove?

**Solution**

$g \circ f = (g \Theta f) \oplus f$

peaks smaller than the domain of the structuring element f will be removed.

**f.**

Compute the open of the given data. Interpret your results. What features are preserved and what features are removed?

**Solution**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 7 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | g |
| y | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| | 0 | | | | | | | 7 | |
| | | x | | | | | | | |

open

the open operator removes all the small peaks in the image

**g.**

Define the top-hat operator for gray-level morphology. What type features will it locate?

**Solution.**

$tophat(g) = g - (g \circ f)$

The $(g \circ f)$ open is beneath g. Hence the tophat operator is useful for finding objects that are peaks on a flat background.

**h**

Define the boundary operator for gray-level morphology.

**Solution**

if g is the image function and f is the structuring function

then $grad(g) = (g \oplus f) - (g \Theta f)$.

**i**.

For the given data compute the top-hat transform of g with f

**Solution**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 1 | 0 | 0 | 8 | 0 | 0 | 0 |
| y | | 0 | 1 | 0 | 0 | 8 | 8 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 8 | 8 | 0 | 0 |
| | | 0 | 0 | 0 | 0 | 8 | 8 | 0 | 0 |
| | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | | | x | | | | | | |

Tophat operation

## 1.8 Exercise

Gray-scale Morphology and Pattern Spectra

a.

Define the cumulative distribution function for gray-scale pattern spectra, F(k)

b.

Define the probability function p(k)

c.

What is the meaning of the term p(k)?

d.

If f the structuring element is

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | ⓪ | 0 |
| 0 | 0 | 0 |

give 2f, 3f

e.

Given the following data for image g, compute the open of g with f , 2f, 3f

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | | | | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | | | | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | 5 | 5 | 5 | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | 5 | 5 | 5 | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | 5 | 5 | 5 | | |
| | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | |

f.

Compute the pattern spectra for g.

**Solution.**

A(0) is the area under the curve of g , $A(g)=\sum_p g(p)$

A(k)=$A(k)=\sum_p g\circ kf$

F(k)=$1-\frac{A(k)-A(\infty)}{A(0)-A(\infty)}$

p(k)=$p(k)=F(k+1)-F(k)=\frac{A(k)-A(k+1)}{A(0)-A(\infty)}$

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

f

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

2f

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

3f

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | | | | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | | | | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | 5 | 5 | 5 | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | 5 | 5 | 5 | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | 5 | 5 | 5 | | |
| | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | |

Open with f

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | | | | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | | | | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | | | | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | | | | | |
| | | 7 | 7 | 7 | 7 | 7 | | | | | | | | |
| | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | |

Open with 2f

$A(\infty) = 0$
A(0)=25*7 + 9*5=175+25=220
A(1)= A(0)=220
A(2)=25*7=175
A(3) = 0
F(0)=0
F(1)=1-(220/220)=0
F(2)=1-(175/220)=1-.7954=.2045
F(3)=1-(0/220)=1
p(0)=(220-220)/220=0
p(1)=(220-175)/220=.2045
p(2)=(175-0)/220=.7945
p(3)=(0-0)/220=0
p(k) is the normalized area of the objects which are of size kf

## 1.9 Exercise

**a.**

Define grayscale geodesic dilation $\delta_g(f)$ where $f \leq g$. Use the unit ball for the structuring element.

**Solution**

If $f \leq g$ then the geodesic dilation of function f with respect to g is $\delta_g(f) = (f \oplus f_B) \wedge g$. The function g is referred to as the mask and the function f is the marker function. The structuring element $f_B$ is the unit ball.

**b.**

Define $\delta_g^n(f)$ and $\delta_g^\infty(f)$

**Solution**

And $\delta_g^2(f) = \delta_g(\delta_g(f))$. The repeated application of the function n times is $\delta_g^n(f) = \underbrace{\delta_g^1 \circ \delta_g^1 \circ \ldots \circ \delta_g^1}_{\text{n times}}(f)$.

One iterates grayscale geodesic dilation of f under g until stability is reached to obtain $\delta_g^\infty(f)$

**c.**

Compute the gray-scale geodesic dilation of $\delta_g^1(f)$ for the given functions.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| g | 7 | 6 | 8 | 9 | 8 | 4 | 2 |
| f | 4 | 4 | 4 | 2 | 4 | 2 | 1 |

**Solution**

| | | | | | | |
|---|---|---|---|---|---|---|
| g | 7 | 6 | 8 | 9 | 8 | 4 |
| f | 4 | 4 | 4 | 2 | 4 | 2 |
| $f \oplus f_s$ | 5 | 5 | 5 | 5 | 5 | 5 |
| $\delta_g^1(f)$ | 5 | 5 | 5 | 5 | 5 | 4 |

**d.**

Compute the gray-scale geodesic dilation of $\delta_g^{\infty}(f)$

**Solution**

| | | | | | | |
|---|---|---|---|---|---|---|
| g | 7 | 6 | 8 | 9 | 8 | 4 |
| f | 4 | 4 | 4 | 2 | 4 | 2 |
| $f \oplus f_s$ | 5 | 5 | 5 | 5 | 5 | 5 |
| $\delta_g^1(f)$ | 5 | 5 | 5 | 5 | 5 | 4 |
| $\delta_g^1(f) \oplus f_s$ | 6 | 6 | 6 | 6 | 6 | 6 |
| $\delta_g^2(f)$ | 6 | 6 | 6 | 6 | 6 | 4 |
| $\delta_g^2 \oplus f_s$ | 7 | 7 | 7 | 7 | 7 | 7 |
| $\delta_g^3(f)$ | 7 | 6 | 7 | 7 | 7 | 4 |
| $\delta_g^3 \oplus f_s$ | 8 | 8 | 8 | 8 | 8 | 8 |
| $\delta_g^4(f)$ | 7 | 6 | 8 | 8 | 8 | 4 |
| $\delta_g^4 \oplus f_s$ | 8 | 9 | 9 | 9 | 9 | 9 |
| $\delta_g^5(f)$ | 7 | 6 | 8 | 9 | 8 | 4 |
| $\delta_g^5 \oplus f_s$ | 8 | 9 | 9 | 9 | 9 | 9 |
| $\delta_g^6(f)$ | 7 | 6 | 8 | 9 | 8 | 4 |