

1. Hierarchical Representations of Images

There are many different types of hierarchical representations of images in the image domain. These are also called image pyramids [Jain, Kasturi, and Schunck, 1995, pp. 89]. One refers to the different levels of the pyramid or tree. The different levels have a different resolution. The highest level would consist of the resolution of the acquired image data. It should be sufficient to capture the smallest details of interest in the image. The next level would be at a lower resolution. As we move to lower resolutions small details in the image are lost. The amount of data is also reduced making analysis at lower levels much faster. The type of a representation in the image domain lends itself to implementing vision systems that use hierarchical models in the scene-domain.

The simplest way to form an image pyramid would be to divide the resolution by powers of two. For example the resolution might be 256 by 256 at the bottom level of the pyramid and 128 by 128 at the next level. The reduction could be accomplished by averaging over a 4 by 4 neighborhood [Jain, Kasturi, and Schunck, 1995, pp. 89] or by wavelet methods [Mallat, 1998]. The following figure indicates this operation. The nodes in the hierarchy represent pixels at a given level.

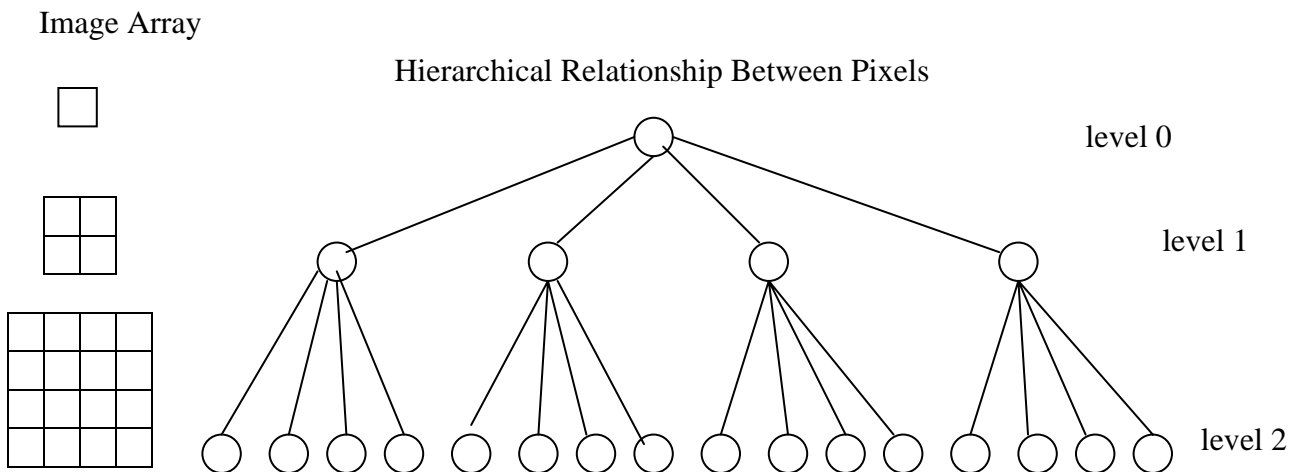


Figure 1. Pyramid Structure

1.1 Picture Trees

Picture trees and quad trees apply more to binary images pyramids [Jain, Kasturi, and Schunck, 1995, pp. 89,94]. The binary image may be a representation of regions or objects in the image. If one is performing segmentation then one divides the image into smaller and smaller regions until all the pixels in the different regions satisfy the uniformity predicate for that region. A picture tree describes this process. A picture tree is a graph structure in the form of a tree that has an arc between two nodes of the graph, R1 and R2, if R2 is contained in R1. R1 is the parent node of R2. In the following example R1 includes R2, R3, R4 and these sets form a partition of R1. These structures are often useful in implementing and describing region splitting and region merging segmentation methods.

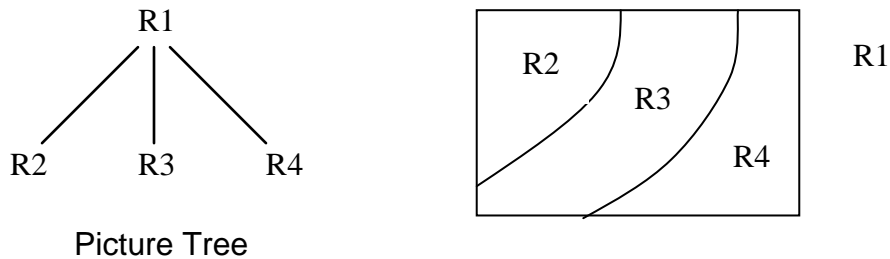


Figure 2. Picture Tree

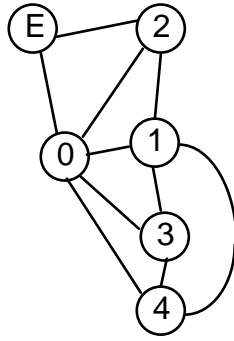
1.2 Region Adjacency Graph (RAG)

A structure often used with the picture trees and other segmentation methods is called the region adjacency graph which gives the region adjacency relations [Pavlidis, 1982, pp. 121]. Region R1 is adjacent to region R2 if there is a pixel in R1 with a 4-neighbor in R2. Given a set of regions, the region adjacency graph is a graph where each node represents a region and an arc is drawn between regions R1 and R2 if they are adjacent. The nodes represent the regions and the arcs represent a common boundary between the regions. After an initial segmentation the RAG may be used to further combine regions based upon their adjacency relationships. A RAG for a partition may be generated in the following manner [Jain, Kasturi, and Schnuck, 1995, pp. 92]. Let $\text{Label}(x,y)$ be an array coding of the region membership information.

1. Perform a raster scan over the coordinate system to find a region
 $r = \text{Label}(x,y)$
2. Let $r_1 = \text{Label}(x_1,y_1)$ be a neighbor of r . If $r \neq r_1$ then draw an arc between r and r_1 in the RAG.

In the following example, the region codes are given in the image grid. Observe that region 0 is a global region.

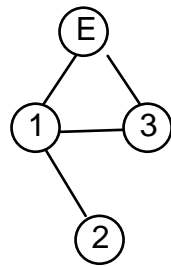
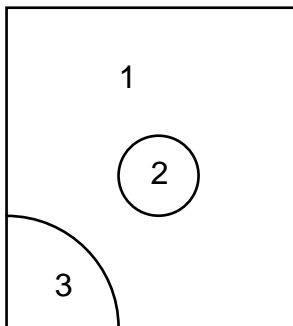
0	0	0	0	0	0
0	1	1	4	4	0
0	1	1	3	0	0
0	1	1	3	0	0
0	0	2	0	0	0
0	0	2	0	0	0



Region Adjacency Graph

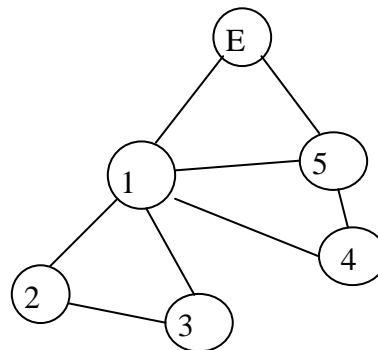
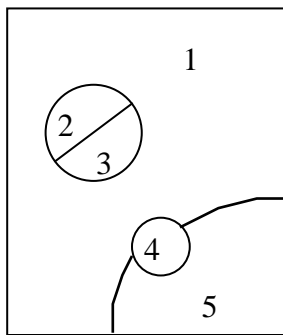
Figure 3. Region Adjacency Graph

The degree of a node is the number of nodes connected to it by an arc. Nodes of high degree often correspond to large or global regions. Nodes of degree 1 correspond to a hole. A cut-node of a graph is a node c such that there are two other nodes a and b in the graph with the property that all paths from a to b go through node c . If a region completely surrounds another region, it will correspond to a cut-node of the RAG.



2 is a hole
1 is a cut-node

Region Adjacency Graph



4 is a transition node

Figure 4. Region Adjacency Graph

Global regions, these are regions which extend over a major part of the image, will tend to have a high degree. Region 1 in the above figure is a global region.

Transition regions have a small degree and should be connected to two nodes of high degree.

The dual of a RAG is a graph where the nodes represent boundaries and the arcs represent regions [Jain, Kasturi, and Schunck, 1995, pp.94].